Short communication

Relations among stability parameters in the stable surface layer: Golder curves revisited

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Abstract

A nomogram was prepared by [Golder, 1972. Boundary Layer Meteorology 3, 47–58] to compute the surface layer parameters in stable conditions. This note revisits the Golder’s curves and examines the methodology underlying their derivation in stable conditions. The inherent limitation in the methodology used for construction of Golder’s curves was also noticed by Trombetti et al. (1986). Surface layer fluxes computed using the parameters derived from modified curves are found to be closer to the turbulence measurements from CASES-99 experiment for stable conditions than those calculated from the [Golder, 1972. Boundary Layer Meteorology 3, 47–58] curves.

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1. Introduction

Golder (1972) proposed a methodology for constructing a nomogram between \( \ln(z/z_0) \) (\( z \) is the height above the ground and \( z_0 \) is the roughness length) and gradient Richardson number \( (R_i) \) for a fixed value of bulk Richardson number \( (R_b) \). This nomogram was widely used for describing the fluxes in the atmospheric models. However, Golder used the derivative of potential temperature in the definition of \( R_b \) rather than the temperature difference in the bulk layer. These two methods give rise to different expressions of \( R_b \) in terms of \( z/L \) (\( L \) is Monin–Obukhov length). Trombetti et al. (1986) also pointed out the discrepancy in Golder’s curves and indicated the conditions for their validity. Golder (1972) suggested taking \( z = \bar{z} \) as the mean geometrical height between the two levels where temperatures are measured and wind speed \( u \) at \( z \). Wang (1981) proposed \( z \) as the upper temperature level whereas Sedefian and Bennet (1980) used \( z = \bar{z} \) and \( u \) at the upper level.

In this note, we revisit Golder’s curves by introducing them first, and then using a systematic approach to construct the modified curves in stable conditions. The surface fluxes computed using the parameters drawn from these curves are compared with those based on turbulence measurements in stable conditions from CASES-99 experiment.

2. Methodology

From similarity theory, under horizontally homogeneous and steady state conditions:

\[
\frac{kz}{u_*} \frac{\partial u}{\partial z} = \phi_m \left( \frac{z}{L} \right),
\]

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\[ k z \frac{\partial \theta}{\partial z} = \Phi_h \left( \frac{z}{L} \right). \] (2)

Here \( k = 0.4 \) is von Karman constant, \( z \) is height above the ground, \( u_* \) is friction velocity, \( \theta_0 \) is friction temperature, \( u \) is mean wind speed, \( \theta \) is potential temperature, \( \Phi_m \) and \( \Phi_h \) are universal functions of the stability parameter \( z/L \), where \( L \) is defined as

\[ L = \frac{u_*^2 T_0}{kg \theta_*}, \] (3)

\( g \) is the acceleration due to gravity and \( T_0 \) is the air temperature.

Integration of Eqs. (1) and (2) gives (Panofsky and Dutton, 1984, p. 134):

\[ u = \frac{u_*}{k} \left[ \ln \frac{z}{z_0} - \Psi_m \left( \frac{z}{L} \right) \right], \] (4)

\[ \theta - \theta_0 = \frac{\theta_u}{k} \left[ \ln \left( \frac{z}{z_0} - \Phi_h \left( \frac{z}{L} \right) \right) \right], \] (5)

where \( \theta_0 \) is the temperature at \( z = z_0 \) and

\[ \Psi_m \left( \frac{z}{L} \right) = \int_{z_0/L}^{z/L} \left( 1 - \Phi_m (\zeta) \right) \frac{d\zeta}{\zeta}, \] (6)

in which \( \zeta = z/L \).

The gradient Richardson number (\( Ri \)) is defined as

\[ Ri = \frac{g}{T_0} \frac{\partial u}{\partial z} \left( \frac{z}{L} \right). \] (7)

This can be expressed as a function of \( z/L \) using Eqs. (1)–(3) as

\[ Ri = \frac{z}{L} \frac{\Phi_h (z/L)}{\Phi_m (z/L)}, \] (8)

In stable conditions, \( \Phi_m \) and \( \Phi_h \) are expressed as linear functions of \( z/L \) (Businger et al., 1971). Golder’s curves are based on the relation

\[ \Phi_m = \Phi_h = 1 + \frac{z}{L}, \] (9)

where \( \beta \) is a constant and \( \Psi_m = \Psi_h \). From Eqs. (8) and (9),

\[ \frac{z}{L} = \frac{Ri}{1 - \beta Ri}. \] (10)

2.1. Golder’s method

Golder (1972) defined the bulk Richardson number (\( Rb \)) as

\[ Rb = \frac{g}{T_0 u_*^2} \frac{\partial \theta}{\partial z} z^2 \] (11)

From Eqs. (7) and (11), \( Rb \) is related to \( Ri \) (Panofsky and Dutton, 1984, p. 143) as

\[ Ri = \frac{Rb}{p^2}, \] (12)

where

\[ p = \frac{\partial \theta}{\partial \ln z} = \frac{z}{u} \frac{\partial \theta}{\partial z}. \] (13)

Golder expressed \( u \) and \( \partial \theta/\partial z \) as a function of \( Ri \) in Eq. (13). Using Eqs. (1), (4) and (10) in (13), \( p \) can be expressed (Panofsky and Dutton, 1984, p. 138) as

\[ p = \frac{\Phi_m (Ri)}{\ln \left( \frac{z}{z_0} - \Psi_m (Ri) \right)}. \] (14)

In stable conditions, \( \Phi_m \) and \( \Psi_m \) in terms of \( Ri \) can be given as

\[ \Phi_m (Ri) = 1 - \frac{1}{1 - \beta Ri}, \] (15)

\[ \Psi_m (Ri) = - \frac{\beta Ri}{1 - \beta Ri}. \] (16)

Eqs. (15) and (16) are valid only if \( K_h/K_m \) is assumed to be 1. Here \( K_h \) and \( K_m \) are eddy diffusivities for heat and momentum respectively. The value of \( \beta \) was taken as 7 by Golder (1972).

Notice that Eq. (12) is a quadratic equation in \( Ri \). This can be solved for the lowest value of \( Ri \) for given values of \( \ln(z/z_0) \) and \( Rb \).

2.2. Alternative procedure

Golder (1972) used the gradient of the potential temperature in the definition of \( Rb \) as shown in Eq. (11). On the other hand, the gradient of the potential temperature should be approximated as the potential temperature difference divided by the layer thickness and accordingly \( Rb \) is defined as

\[ Rb = \frac{g}{T_0 u_*^2} \frac{(\theta - \theta_0)}{u^2} z. \] (17)

Using Eqs. (4) and (5) along with Eqs. (3) and (9) in Eq. (17), we get

\[ Rb = \frac{z}{L} \left[ \ln \left( \frac{z}{z_0} \right) + \beta \frac{z}{L} \right]^{-1}. \] (18)

This gives

\[ \frac{z}{L} = \frac{Rb \ln \left( \frac{z}{z_0} \right)}{1 - \beta Rb}. \] (19)

For given values of \( \ln(z/z_0) \) and \( Rb \), Eq. (19) is used to compute \( z/L \) which in turn is utilized to calculate \( Ri \).
from Eqs. (8) and (9) as

\[ Ri = \frac{z/L}{1 + \beta z/L}. \]  

(20)

Notice that in the expressions of \( Rb \) and \( \Psi_m \), we assumed \( z \gg z_0 \), and thus, \( z_0 \) compared to \( z \) has been ignored in order to minimize the degrees of freedom for constructing nomogram for known values of \( \ln(z/z_0) \) and \( Rb \). Since the value of \( Rb \) is assumed to be known from the observations, we have taken \( z \) as the upper level at which the measurements of temperature and wind speed are available, in both Golder’s and modified methodology. The same approach has also been adopted by Trombetti et al. (1986).

3. Results and discussion

In Golder’s formulation, for given values of \( Rb \) and \( \ln(z/z_0) \), the lowest value of \( Ri \) was computed from the quadratic equation deduced from Eq. (12). These values of \( Ri \) were used to construct a nomogram

![Fig. 1. Ri as a function of ln(z/z0) and Rb: (a) reproduced from Golder (1972), Boundary Layer Meteorology, and (b) with present approach. Isopleths of Rb have been multiplied by 100.](image)

![Fig. 2. Evaluation of (a) friction velocity and (b) heat flux computed from Golder’s nomogram (△) and modified nomogram (●) with CASES-99 data (○).](image)
between $\ln(z/z_0)$ and $R_i$ for a fixed value of $R_b$. This reproduces the nomogram given by Golder (1972) in Fig. 1a.

For constructing a nomogram based on Eq. (14) by expressing $p$ explicitly as a function of $z/L$, we express Eq. (12) in terms of $z/L$ by using Eqs. (1), (4), (9) and (20), as

$$R_b = \frac{z}{L} \left[ 1 + b \frac{z}{L} \left[ \ln \left( \frac{z}{z_0} \right) + \beta \frac{z}{L} \right] \right]^{-2} \tag{21}$$

This is a second-degree polynomial in $z/L$. Trombetti et al. (1986) have also expressed $R_b$ as a function of $z/L$, which reduces to a second degree polynomial in $z/L$ for the linear relation (9) for $\phi_m$. The Eq. (21) will have two values of $z/L$ for a given value of $R_b$ and $\ln(z/z_0)$. The lowest value of $z/L$ computed from Eq. (21) is utilized for calculating $R_i$ from Eq. (20). This method leads to the nomogram as given by Golder (1972). However, Golder’s expression for $R_b$ in terms of $z/L$, Eq. (21), is different from that derived in our approach, Eq. (18). The discrepancy, which arises due to the incorrect definition of $R_b$ used by Golder (1972) in constructing the nomogram, was also noticed by Trombetti et al. (1986).

Figs. 1a and b shows the variation of $\ln(z/z_0)$ with $R_i$ for a fixed $R_b$. Golder’s (1972) nomogram is shown in Fig. 1a and the nomogram constructed on the basis of the modified procedure is given in Fig. 1b. The curves are steeper with the modified method than those obtained from Golder’s method.

In Fig. 1, $\beta = 7$ is taken following Golder (1972). However, the well-established value of $\beta = 5$ as proposed by Dyer (1974) is adopted for computation of results in the rest of this paper. Turbulence measurements from CASES-99 (Mahrt et al., 2001) in stable conditions for $R_b < 0.15$ (Sharan et al., 2003) are used for validating the fluxes computed using the parameters obtained from the nomogram. Wind and temperature measurements at 10 m level are used. The temperature at $z_0$ is obtained by extrapolating temperatures measured at 10 and 1.5 m. Friction velocity (Fig. 2a) and heat flux (Fig. 2b) computed from the modified nomogram are found to be closer to the turbulence observations than those obtained from Golder’s (1972) nomogram. This is confirmed quantitatively by computing the Normalised mean square error (NMSE). The NMSE values are 0.096 and 0.05 for $u_*$ computed from Golder and modified nomogram respectively. The corresponding value of NMSE for heat flux reduces from 0.498 to 0.112 with the modified curves. In addition, Figs. 3 and 4 reveal that the surface layer parameters computed from modified curves are found to be closer to the observations than those obtained from Golder’s curve. The fluxes computed using the procedure outlined in Trombetti et al. (1986) are found to be almost same to those obtained here based on the assumption that $z$ is taken as the upper level, $z_0$ as the lower level and $u = u(z)$.

For comparison with turbulence data, the commonly used value of $\beta = 5$ is taken. However, by using the value of $\beta = 7$ following Golder (1972), the NMSE values are 0.191 and 0.069 for $u_*$ computed from Golder and modified nomograms respectively. The
corresponding values for the surface heat flux are 1.017 and 0.139.

Both nomograms are based on the use of linear functions of $z/L$ for non-dimensional wind and temperature profiles. However, the applicability of the linear function is limited to $Rb < 1/\beta$ (Sharan et al., 2003). Much larger values of $Rb$ are often observed in weak wind stable conditions.

4. Conclusions

This note revisits the Golder’s (1972) curves relating $Ri$ and $\ln(z/z_0)$ for fixed values of $Rb$ and examines the methodology underlying their derivation. New curves derived from the modified method are shown to differ somewhat from those given by Golder. Both qualitative and quantitative analysis of the surface layer fluxes derived from the two methods in relation to the observed turbulent fluxes in stable conditions from CASES-99 experiment show that the modified method performs better.

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